1. A coin is tossed twice. Alice claims that the event of two heads is at least as likely if we know that the first toss is a head than if we know that at least one of the tosses is a head. Is she right? Does it make a difference if the coin is fair or unfair? How can we generalize Alice’s reasoning?

2. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely. (a) Find the probability that doubles are rolled. (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled. (c) Find the probability that at least one die roll is a 6. (d) Given that the two dice land on different numbers, find the conditional probability that at least one die roll is a 6.

3. You enter a chess tournament where your probability of winning a game is 0.3 against half of the players (call them type 1), 0.4 against a quarter of the players (call them type 2), and 0.5 against the remaining quarter of the players (call them type 3). You play a game against a randomly chosen opponent. (a) What is the probability of winning? (b) Suppose that you win. What is the probability that you had an opponent of type 1?

4. The Monty Hall Problem. This is a much discussed puzzle, based on an old American game show. You are told that a prize is equally likely to be found behind any one of three closed doors in front of you. You point to one of the doors. A friend opens for you one of the remaining two doors, after making sure that the prize is not behind it. At this point, you can stick to your initial choice, or switch to the other unopened door. You win the prize if it lies behind your final choice of a door. Consider the following strategies: (a) Stick to your initial choice. (b) Switch to the other unopened door. (c) You first point to door 1. If door 2 is opened, you do not switch. If door 3 is opened, you switch. Which is the best strategy?

5. Consider the following medical diagnosis problem in which there are two alternative hypothesis: (1) The patient is COVID-19 positive, and (2) that the patient does not. The available data is from a particular rapid antigen based test kit with two possible outcomes: ⊕ (positive), and - (negative). We have the prior knowledge that over the entire population only 0.08 has this disease. Furthermore, the lab test is only an imperfect indicator of the disease. The test returns a correct positive result in only 90% of the cases in which the disease is actually present, and a correct negative result in only 97% of the cases in which the disease is not present. In other cases, the test returns the opposite result. Suppose we observe a new patient for whom the lab returns a positive result. Given this information,

1. Should we diagnose the person as having COVID-19 or not? (What is the probability of the person having COVID-19?)

2. Assuming that the prior knowledge that the entire population has this disease is 0.6, what is the probability of the person having the disease?